Evidence of α particle condensation in ^{12}C and ^{16}O and Nambu-Goldstone boson

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Abstract

It is suggested that direct evidence of Bose-Einstein condensation of α particles is obtained by observing a phase mode (Nambu-Goldstone boson) with long wavelength even when characteristic features such as superfluidity is diffucult to observe. For the 7.65 MeV 0_2^+ Hoyle state in ^{12}C and 15.1 MeV 0^+ state in ^{16}O, which are candidates for an α particle condensate, it is suggested that the emergent band head 0^+ state of the $K = 0_2^+$ rotational band with a very large moment of inertia is considered to be a Nambu-Goldstone boson.

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The 0_2^+ state in ^{12}C at excitation energy E_x=7.65 MeV, the Hoyle state, is a key state for nucleosynthesis, the evolution of stars and the emergence of life. The existence of this resonant state just above the α threshold was predicted by Hoyle to explain the enhanced triple α reactions needed to understand the nucleosynthesis of ^{12}C. In the last 60 years, the understanding of the Hoyle state has been deepened by the study of the α cluster structure of ^{12}C. However, its interpretation has changed considerably. Morinaga [1] proposed that the Hoyle state is a band head state with a linear chain structure of three α particles. The three α structure of ^{12}C was most thoroughly investigated by Uegaki et al. [2] in their pioneering work, which showed that the Hoyle state has a dilute structure in a new "α-boson gas phase" and clarified the systematic existence of a "new phase" of the three α particles above the α threshold.

Inspired by the observation of Bose-Einstein condensation of cold atomic gas clusters, much attention has been paid to investigate whether Bose-Einstein condensation of α particles occurs in light nuclei where α cluster structure widely exists. The three α particle system of ^{12}C, especially the dilute properties of the Hoyle state due to α particle condensation, has been studied extensively by many authors [3, 4, 5, 6, 7, 8, 9, 10, 11]. It is now evident theoretically and experimentally that the Hoyle state has a larger radius compared with the ground state and has a dilute matter distribution.

In order to get direct experimental evidence that the Hoyle state is a condensate, Raduta et al. [10] observed three α particles with the same energy emitted simultaneously from the Hoyle state and suggested that the three α particles were sitting in the lowest $0s$ state. On the other hand, in a similar coincidence experiment Manfredi et al. [11] skeptically set an upper limit of a probability of 0.45%, which confirms the previous result by Freer et al. [12]. In spite of much effort, firm experimental evidence of Bose-Einstein condensation such as superfluidity or a vortex has not been found for the Hoyle state.

The purpose of this paper is to suggest that evidence of α particle condensation can be provided by the observation of the Nambu-Goldstone (NG) boson caused by the spontaneous symmetry breaking of the global phase symmetry. We suggest that for ^{16}O a bandhead 0^+ state predicted at 16.6 MeV that accompanies a $K = 0_2^+$ band with very large moment inertia is a NG boson collective zero mode associated with the condensation of four α particles of the 0^+ state at 15.1 MeV. We also suggest in ^{12}C that a 0_3^+ state and a rotational band built on it should emerge slightly above the Hoyle state as a NG boson if the Hoyle state is a condensate of three α particles. We show an experimental candidate for the NG boson state in ^{16}O and ^{12}C.

If the concept of α particle condensation persists in nuclei, the four α particle system of ^{16}O is another good

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candidate. Tohsaki et al. [3] conjectured that the 0_3^+ state at 11.26 MeV in ^{16}O, which is located 3.18 MeV below the four α threshold energy, may be an α particle condensate because it is well represented by a Bose wave function with a dilute four α particle structure. Funaki et al. [13] investigated 0^+ states in ^{16}O using a four α cluster model in the bound state approximation and suggested that the 0^+ state at 15.1 MeV is probably an α condensed state. Ohkubo and Hirabayashi [14] investigated not only the 0^+ states but also the so-called four α linear chain states with higher spins [15, 16] from the viewpoint of the unified description of quasi-bound states and α scattering by analyzing the α+^{12}C elastic scattering and α+^{12}C(0_2^+) inelastic scattering using a coupled channel method. They showed that the so-called four α linear chain states can be understood as a $K = 0_2^+$ band (Fig. 1) with the α+^{12}C(0_2^+) configuration. They suggested that the observed 15.1 MeV 0^+ state is a candidate for the four α particle condensate in a completely different quantum state under Bose statistics.

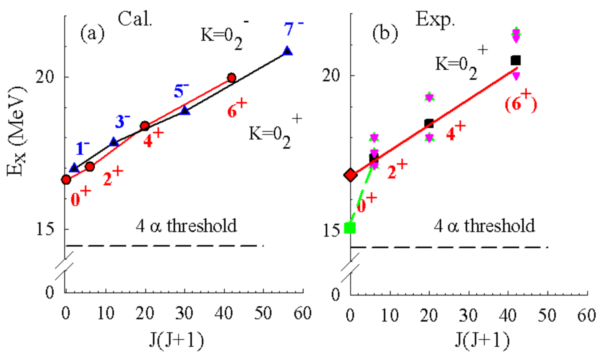

Figure 1: (a) The calculated $K = 0_2^+$ and $K = 0_2^-$ rotational bands with the α+^{12}C(0_2^+) cluster structure in ^{16}O near the four α threshold [14]. (b) The experimental rotational band states with α cluster structure taken from Ref.[14, 18]. The centroid of each of the spin states is shown by a black square. The candidate 15.1 MeV 0^+ state of four α condensate is displayed by a green square. The lines are to guide the eye.

If the 15.1 MeV 0^+ state is a special state of a Bose-Einstein condensate of four α particles, the state can be described by a macroscopic wave function in the Ginzburg-Landau theory, $\Psi = |\Psi| e^{i\theta}$ where $|\Psi|$ is the order parameter. In a condensate, continuous global phase symmetry in gauge space is spontaneously broken with $|\Psi| \neq 0$ in the NG phase and two collective modes appear, i.e., a massless (zero energy with a infinite wavelength) phase (θ) mode (NG boson) and a finite mass amplitude ($|\Psi|$) mode (Higgs boson) [17]. Spontaneous symmetry breaking is ubiquitous in physical systems and in nature [19]. For example, for infinite systems, in superconductors the NG mode (in which the NG boson has been eaten by the plasmon [20]) and the Higgs mode [21] have been observed. As seen in Fig. 2, in particle physics the massless pion and the amplitude mode σ meson emerge as a collective mode from the vacuum due to the chiral condensation of the vacuum [17]. We note that a band (Regge trajectory) of the states, in which orbital angular momentum of the relative motion between a quark and an anti-quark is excited, is built on top of the NG boson pion. The spectra including the radially excited states are explained well by the quark model [23]. For finite systems a Nambu-Goldstone boson as well as a Higgs mode boson have been observed in superfluid nuclei as a pairing rotation and a pairing vibration, respectively [24]. For Bose-Einstein condensed cold atom gases with finite number of particles in a finite volume, the existence of a Nambu-Goldstone boson has been shown theoretically from the Ward-Takahasi identity in quantum field theory [25, 26].

The appearance of the $K = 0_2^+$ band in Fig. 1(a) predicted in Ref.[14] just above the 15.1 MeV 0^+ state may be understood from the viewpoint of spontaneous symmetry breaking as follows: If the 15.1 MeV 0^+ state is a condensate, a NG boson zero energy collective mode should appear. Considering that (1) the 0^+ at 16.6 MeV is a first 0^+ state above the condensed vacuum (15.1 MeV 0^+ state) with the very small excitation energy of 1.5 MeV (almost massless) and (2) the very large moment of inertia of the band built on it, which is due to a collective motion with the largest dimension that the system allows (long wavelength), this state may be regarded as the emergence of the NG boson caused by the spontaneous symmetry breaking of the global phase due to the four α particle condensation. This NG state is a collective state with enlarged structure of four α particles with dilute property due to its long wavelength nature, i.e. deformed orthogonal to the spherical vacuum 15.1 MeV 0^+ in the $0s$ states.

In fact, in Ref.[14] it was shown that the 0^+ resonant state predicted at 16.6 MeV has a loosely coupled α+^{12}C(0_2^+) cluster structure with large deformation. As shown in Fig. 1 (a), their calculated resonances with spin 2^+, 4^+ and 6^+ are located on the J(J+1) line forming a rotational band. The predicted negative parity band $K = 0_2^-$ [14] is considered as a parity-doublet partner of the $K = 0_2^+$ band with almost the same moment of inertia. The very large moment inertia of the $K = 0_2^+$ band stems from the NG boson nature of the band head 0^+ state with a massless long wave collective motion. It is noted that the calculated resonances of the $K = 0_2^+$ band correspond well with the experimental α cluster

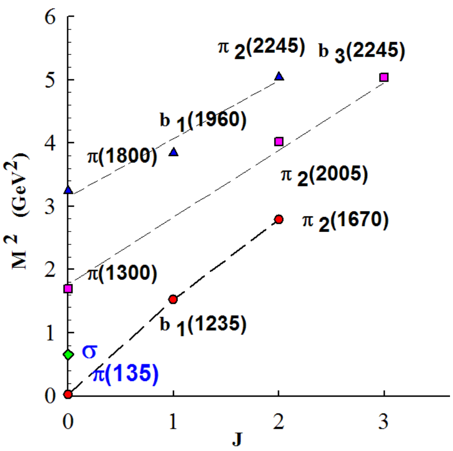

Figure 2: The experimental spectra of the NG boson pion, and the orbital and radial excited states of the pion are displayed together with the Higgs boson σ meson. The lines are to guide the eye. Data are from Ref.[22].

states observed in the four α coincidence experiments [15, 16].

Before proceeding to the experimental candidate for the NG boson 0^+ state, we would like to emphasize that the 15.1 MeV 0^+ state should not considered as a member state of the $K = 0^+_2$ band built on the NG 0^+ state. The intrinsic structure of the 15.1 MeV 0^+ state is quite different in nature from the deformed NG boson 0^+ state and its associated rotational band states. The NG boson state is a logical consequence of the existence of the 15.1 MeV state that spontaneously violates the global phase symmetry due to condensation. While the 15.1 MeV 0^+ state is spherical with Bose statistics in the $0s$ state, the $K = 0^+_2$ rotational band states are deformed in a local condensate state in which the three α particles are condensed forming the Hoyle state [14]. If we connect the 15.1 MeV 0^+ state and the 2^+ state (Fig. 1(b)), the estimated moment of inertia appears drastically reduced from that of the $K = 0^+_2$ rotational band as discussed in Ref.[14].

According to our interpretation, the existence of the NG boson 0^+ state predicted at 16.6 MeV is logically very important. However, no 0^+ state has been reported around here in the literature. It may not be easy to observe the NG boson because of a large decay width of more than 1 MeV. Very recently Itoh et al. reported [18] that they have observed new broad resonant 0^+ states at 16.8 MeV and 18.8 MeV. The 16.8 MeV state with an α cluster structure of a width of about 1 MeV (a diamond in Fig. 1(b)) agrees precisely with the theoretically predicted energy 16.6 MeV and the α width 1.1 MeV (Fig. 1(a)) [14]. This strongly supports the present interpretation. The 18.8 MeV 0^+ state may correspond to the 0^+_4 state around 10 MeV in Fig. 3.

Now we come to the α particle condensation of three α particles in ^{12}C. According to the above interpretation, the understanding of the $K = 0^+_2$ rotational band built on the Hoyle state is drastically changed. Morinaga [1] interpreted that the Hoyle state and the 10.3 MeV 2^+ state form a rotational band with a very large moment of inertia, which leads to the three α linear chain model. Recently 2^+_2 (9.6 MeV) [27, 28, 29] and 4^+_1 (13.3 MeV) [30] states above the Hoyle state have been observed. The 2^+_2 state has been considered to be a rotational band state built on the Hoyle state [31]. If the Hoyle state is a condensate of three α particles, a NG boson zero energy collective mode should appear just above it. The NG boson is massless in principle, however, in nature its energy is not always exactly zero as seen in the pion case. If the NG state has zero energy, the 2^+_2 (9.63 MeV) and 4^+_1 (13.3 MeV) above stand on the Hoyle state. In this case it is unlikely that the 2^+_2 and 4^+_1 states are rotational member states because the condensate Hoyle state is spherical with the three α particles sitting in the 0s state. On the other hand, if the NG boson mode energy is not exactly zero as in the case of ^{16}O, another 0^+ state emerges just above the Hoyle state. Because the NG 0^+ state is not spherical, a rotational band should appear. The observed 2^+_2 (9.63 MeV) and 4^+_1 (13.3 MeV) states are considered to be a member of this rotational band. The band head NG boson 0^+ state and the 2^+ and 4^+ states should have the same α cluster configuration in nature, mostly a loosely coupled α+^{8}Be configuration created by lifting an α particle from the vacuum (Hoyle state). As shown in Fig. 3, this is an analog of the loosely coupled α+^{12}C(0^+_2) structure built on the NG boson 0^+ state predicted at 16.6 MeV in ^{16}O. The intraband $B(E2)$ transitions should be very large if measured because of a large deformation, i.e. large moment of inertia. On the other hand the $B(E2)$ transition from the 2^+_2 to the Hoyle state should be much smaller than that from the intraband $B(E2{:}2^+_2 \to 0^+_3)$.

The results of the *ab initio* calculation of ^{12}C in the bound state approximation in Ref.[32] seems to support the present interpretation theoretically. The calculated 0^+_3, 2^+_2 and 4^+_1 states have the α+^{8}Be intrinsic configuration and the intraband transition $B(E2)$ values are very large, 600 and 310 (e^{2}fm^4) for the transitions $4^+_1 \to 2^+_2$, $2^+_2 \to 0^+_3$, respectively. On the other hand the $B(E2)$ transition from the 0^+_3 to the Hoyle state is one third, 100 (e^{2}fm^4). Kurokawa and Katō emphasized [31, 33] the importance of treating the unbound resonance states in this energy region carefully under the correct three-body

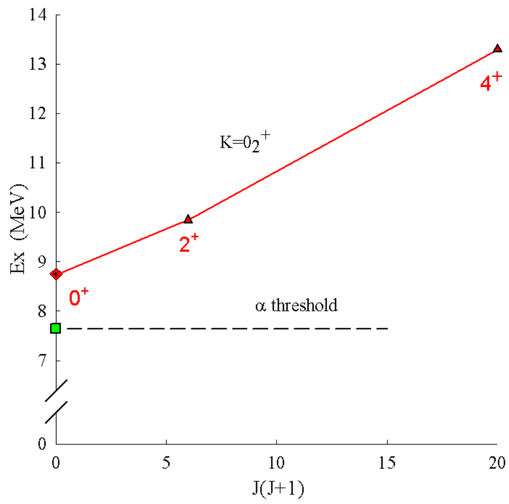

Figure 3: The experimental states with the α cluster structure in ^{12}C: the 0_3^+ state [18] (red diamond) of the Nambu-Goldstone boson, the 2^+ [28] and 4^+ [30] states (red triangle) of the $K = 0_2^+$ rotational band. The α condensate Hoyle state 0_2^+ (green square) and the 0_4^+ state (red circle) [18] are also shown. The line is to guide the eye.

resonance condition and found broad resonance states, 0_3^+ at E_x=8.95 MeV with Γ=1.48 MeV (only 1.66 MeV above the α threshold) and 0_4^+ at E_x=11.87 MeV with Γ=1.1 MeV Ref.[32]. They suggested that this 0_3^+ state may be a higher nodal state, in which radial motion between ^{8}Be and the α particle is excited [33]. We note this α cluster 0_3^+ state is located below the newly observed 2_2^+ (9.63 MeV) state and may correspond to our NG boson 0^+ state because (1) the excitation energy 1.28 MeV from the Hoyle state is small, which is similar to the 1.5 MeV of the ^{16}O case (massless) and (2) this 0_3^+ state forms a $K = 0_2^+$ rotational band with the 2_2^+ (9.63 MeV) and the 4_1^+ (13.3 MeV) states with a large moment of inertia similar to the ^{16}O case (long wavelength).

In the literature no such a 0_3^+ state with a zero-mode nature has been observed just above the Hoyle state. The existence of such a 0_3^+ state will be important in the nucleosynthesis of ^{12}C from triple α particle collisions. From this viewpoint, if we look into the excitation energy of the observed three states carefully, the Hoyle state seems to be located slightly below the energy expected by extrapolating the rotational band of the 2_2^+ (9.63 MeV) and 4_1^+ (13.3 MeV) states. The estimated moment of inertia connecting the Hoyle state and the 2_2^+ (9.63 MeV) state is about 80 % of that estimated from the 2_2^+ (9.63 MeV) and the 4_1^+ (13.3 MeV) states. This trend is similar to the four α case discussed before and seems to suggest the possible existence of a third 0^+ state from the experimental side, because its existence slightly lowers the position of the Hoyle state due to quantum orthogonality. Therefore a third 0^+ state is expected to exist slightly above the position extrapolated from the $J(J+1)$ line of the rotational band for the 2_2^+ (9.63 MeV) and 4_2^+ (13.3 MeV) states. If we look into the experimental data of the isoscalar strength distribution of ^{12}C(α,α') in Fig. 8(a) of Ref.[28], there is a peak in the excitation energy at around 8-9 MeV. This seems to suggest the existence of a new 0_3^+ state at around 9 MeV just above the Hoyle state. Very recently Itoh et al. [18] observed the 0_3^+ and 0_4^+ states with an α cluster structure at 9.04 MeV (width 1.45 MeV) and 10.56 MeV (width 1.42 MeV), respectively. The existence of the 0_3^+ state just above the Hoyle state on which a $K = 0_2^+$ band is built has logically the same structure as seen in the ^{16}O case, which gives strong support to the 0_3^+ state being a NG boson collective mode state. The emergence of the NG boson is strong evidence that the Hoyle state is a condensate of three α particles. The structure of Fig. 3 resembles the ^{16}O case in Fig. 1(b). It is interesting to explore whether a higher spin state built on the 0_4^+, which could be a candidate for the band corresponding to π(1300) in Fig. 2, exists. Kurokawa and Katō [33] predict a 2^+ state on the observed 0_4^+ [18], which forms a $K = 0_3^+$ band with almost the same moment of inertia as the $K = 0_2^+$ band.

We note that a dilute gaseous property with a large radius is not evidence for condensation of α particles without firm evidence such as superfluidity (superconductivity), a vortex or a NG boson. Even a non-condensate state has a gas-like dilute density distribution due to the threshold effect. For example, the 3_1^- state in ^{12}C, which is not a well-developed α cluster state, has a large radius comparable to the Hoyle state due to the threshold effect [9]. The 0_3^+, which is interpreted to have a dominant $[^8\mathrm{Be}(2^+)\otimes\ell{=}2]_{J=0}$ structure in Ref.[2] and a linear chain-like structure in Ref.[32], also has a much larger radius than the Hoyle state. The dilute property of a gaseous state of an α particle system arises for the following four reasons: (1) the quantum α particle condensation in momentum space, which brings about spatial diluteness due to the uncertainty principle, (2) the NG boson state, which causes diluteness because of its long wavelength nature, (3) the threshold effect by which the wave function extends near and above the Coulomb barrier and (4) the Higgs boson state, which appears above the NG boson state, and can be dilute because of the collectivity of the order parameter $|\Psi|$ and the threshold effect. The present mechanism is general and the observation of a NG boson collective mode with long wavelength (large moment of inertia) will be use-

ful for the confirmation of an α particle condensate near the threshold energy.

To summarize, we have investigated α particle condensation in ^{16}O and ^{12}C from the viewpoint that evidence of α particle condensation may be provided by the observation of the NG boson caused by the spontaneous symmetry breaking of the global phase symmetry of a condensate. In ^{16}O the 0^+ state at 15.1 MeV is shown to be a condensate because the band head 0^+ state at 16.6 MeV just above the 15.1 MeV 0^+ state is considered to be a NG boson zero mode state. The $K = 0_2^+$ band built on top of it has a loosely coupled α+^{12}C(0_2^+) cluster structure corresponding well with experimental observation. In ^{12}C we showed that the newly observed 2_2^+ (9.63 MeV) and 4_2^+ (13.3 MeV) α cluster states above the Hoyle state are considered to be a rotational band built on the NG boson 0^+ state. This emerges as a logical consequence of the Hoyle state being a three α condensate state. It is interesting to investigate theoretically and experimentally the emergence of the α condensate and the associated Nambu-Goldstone collective mode in other nuclei.

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